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COMMENT

A comment on 'An important equation for the Anderson model'

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Abstract. We point out that a recent claim by Teng [Teng B 1995 *J. Phys.: Condens. Matter* **7** 867] to have obtained a simple and exact solution of the single impurity Anderson model is incorrect.

In a recent paper, Teng [1] claims to have solved the single impurity Anderson model (AM) [2] exactly; and, for the symmetric case, to have obtained an exact analytical expression for the ground-state energy which coincides with that of the Bethe *ansatz* solution [3, 4]. If true, this would represent a significant advance. We point out, however, that none of these claims is correct, via the following points.

(i) Teng's equation (16) for the ground state energy does not coincide with the Bethe *ansatz* solution (obtained numerically in [5]), or with the perturbation result of Yamada [6] which is exact up to fourth order in the reduced interaction strength, $\tilde{U} = U/\pi\Delta$. The former may be verified by direct numerical comparison with [5]. The latter is immediate: expand Teng's equation (16) perturbatively in $\tilde{U}$, and compare directly with Yamada [6] (see also Hewson's book [7]). Excepting the trivial leading order term $-\tilde{U}/4$, none of the terms in the perturbation expansion is reproduced correctly by Teng's result. That equation (16) reproduces Yamada's result quite well numerically up to $\tilde{U} \sim 2.5$ (figure 1 of [1]) is due simply to the dominance of the leading order term in the perturbation expansion.

(ii) Teng's equation (11) for the impurity Green function $G^{(1)}_{dd;\sigma}$ does not contain any hint of a Kondo resonance, the sole temperature dependence of the resultant Green function arising trivially from that of the mean occupation number $\langle n_{d-\sigma}\rangle$. In fact, for the particle-hole symmetric case considered explicitly where $\langle n_{d-\sigma}\rangle = \frac{1}{2}$ for all T, Teng's result for $G^{(1)}_{dd;\sigma}(\omega)$ is entirely dependent of temperature.

From this alone it is clear that Teng's solution is not exact. The underlying reason is that the 'important equation' in question is not exact, but approximate. In fact, as is clear from the structure of equation (11) for $G^{(1)}_{dd;\sigma}(\omega)$, the physical content of Teng's theory is that of an alloy approximation wherein the effective energy for a σ-spin electron in the impurity can take the values ϵ_d and $\epsilon_d + U$ with probability $(1 - \langle n_{d-\sigma}\rangle)$ and $\langle d_{n-\sigma}\rangle$ respectively.

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